1 (a). The answer is 1, since

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{2^n}{3^{n+1}} &= \sum_{n=0}^{\infty}\frac{1}{3}\cdot\frac{2^n}{3^n}\\ &= \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n\\ &= \frac{1}{3}\frac{1}{1-\frac{2}{3}} \quad \text{(geometric series)}\\ &= 1.\end{aligned}$$

1 (b). The series diverges by the Divergence Test, since $\lim_{n\to\infty}\sqrt[n]{n}=1$ and so the limit

$$\lim_{n\to\infty}\frac{(-1)^n}{\sqrt[n]{n}} \quad \text{DNE.}$$

2 (a). The answer is 8/3, since

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{1+(-1)^n}{2^n} &= \sum_{n=0}^{\infty}\frac{1}{2^n}+\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}\\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n+\sum_{n=0}^{\infty}\left(\frac{-1}{2^n}\right)^n\\ &= \frac{1}{1-\frac{1}{2}}+\frac{1}{1-\frac{-1}{2}}\\ &= 2+\frac{2}{3}\\ &= \frac{8}{3}.\end{aligned}$$

2 (b). The answer is 3/2. Note that (e.g. by partial fractions)

$$\frac{2}{n(n+2)}=\frac{1}{n}-\frac{1}{n+2}.$$

Therefore,

$$\begin{aligned}\sum_{n=1}^{N}\frac{2}{n(n+2)} &= \sum_{n=1}^{N}\left(\frac{1}{n}-\frac{1}{n+2}\right)\\ &= \left(\frac{1}{1}-\frac{\not 1}{\not 3}\right)+\left(\frac{1}{2}-\frac{\not 1}{\not 4}\right)+\left(\frac{\not 1}{\not 3}-\frac{\not 1}{\not 5}\right)+\left(\frac{\not 1}{\not 4}-\frac{\not 1}{\not 6}\right)+\cdots+\left(\frac{\not 1}{\not N}-\frac{1}{N+2}\right)\\ &= 1+\frac{1}{2}-\frac{1}{N+2}\\ &= \frac{3}{2}-\frac{1}{N+2}.\end{aligned}$$

Taking $N\to\infty$, we obtain

$$\sum_{n=1}^{\infty}\frac{2}{n(n+2)}=\frac{3}{2}.$$