1 (a). Absolutely convergent. Notice that

$$\sum_{n=1}^{\infty}\left|\frac{\sin(10n^2)}{n\sqrt{n}}\right| \leq \sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}.$$

The last series converges since it is the p-series with $p = 3/2 > 1$. By the Comparison Test the series

$$\sum_{n=1}^{\infty}\left|\frac{\sin(10n^2)}{n\sqrt{n}}\right|$$

is also convergent. This shows absolute convergence.

1 (b). Conditionally convergent. First, notice that

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\sqrt{n^2+1}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+1}}$$

and that

$$\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty}\sqrt{\frac{n^2}{n^2+1}} = \sqrt{\lim_{n\to\infty}\frac{n^2}{n^2+1}} = 1.$$

Taking $b_n = \frac{1}{n}$ in the Limit Comparison Test, we see that since the series

$$\sum_{n=1}^{\infty}\frac{1}{n}$$

is divergent, the series

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+1}}$$

must also be divergent. This shows that

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n^2+1}}$$

is *not* absolutely convergent. It remains to show that it converges conditionally. By the Alternating Series Test it suffices to show that the sequence

$$\frac{1}{\sqrt{n^2+1}}$$

decreases monotonically to 0. But this is obvious since the denominator $\sqrt{n^2+1}$ increases to ∞.

1 (c). Absolutely convergent. To apply the Root Test, we compute

$$\lim_{n\to\infty} \sqrt[n]{\frac{(1+\frac{1}{n})^n}{e^n}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{e} = \frac{1}{e}.$$

Since $\frac{1}{e} < 1$, the series

$$\sum_{n=1}^{\infty} \frac{(1+\frac{1}{n})^n}{e^n}$$

is absolutely convergent.

1 (d). Absolutely convergent. To apply the Ratio Test, we compute

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{\left(\frac{10^{n+1}}{(n+1)!}\right)}{\left(\frac{10^n}{n!}\right)} = \lim_{n\to\infty} \frac{n!}{(n+1)!} \cdot \frac{10^{n+1}}{10^n} = \lim_{n\to\infty} \frac{1}{n+1} \cdot 10 = 0.$$

Since $0 < 1$, the series

$$\sum_{n=0}^{\infty} \left| \frac{(-10)^n}{n!} \right|$$

is convergent. Therefore the original series is absolutely convergent.

Bonus. Divergent, by the Integral Test.