

Math 231 Practice Midterm 1

1. Let

$$f(x) = \begin{cases} x + 2 & x < -1 \\ 2 & x = -1 \\ x^2 & -1 < x < 1 \\ 1 & x \geq 1. \end{cases}$$

- Sketch the graph of the f .
- Determine the values of a for which $\lim_{x \rightarrow a} f(x)$ exists.
- Determine the values of a at which f is continuous.
- Determine the values of a for which $f'(a)$ exists.
- Find $f(f(-3))$.

2. Evaluate the limit, if it exists.

(a) $\lim_{x \rightarrow -1} \frac{\sqrt{x+2}}{x^2+1}$

(b) $\lim_{t \rightarrow 1} \frac{3t^2 - 3t}{2t^2 - 2}$

(c) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

(d) $\lim_{s \rightarrow -\infty} \frac{2 + s - 3s^2}{3 + 4s^2}$

(e) $\lim_{y \rightarrow \infty} \frac{2}{\sqrt{y^2 + 1} - y}$

3. Let $f(x) = x^2$.

- Use the *definition* of derivative to find $f'(1)$.
- Find an equation of the tangent line to the graph of f at $(1, 1)$.
- Find the linear approximation of $f(x)$ at $a = 1$.
- Use (c) to approximate $(1.025)^2$.

4. Find the *second* derivative of the function.

(a) $f(x) = x^3 + 2\sqrt{x} - \frac{1}{x^2}$

(b) $g(\theta) = 2\theta \sin \theta$

(c) $h(t) = \sqrt{t^2 + 1}$

5. Find $\frac{dy}{dx}$ at $(x, y) = (-2, -2)$ by implicit differentiation.

$$y^2 - x^3 = 3xy$$

6. Two cars start moving from the same point. One travels south at 30 mi/h and the other travels west at 40 mi/h. At what rate is the distance between the cars increasing after an hour?