Math 231 Practice Midterm 2

1. Let $f(x) = x^4 - 2x^3 + x^2$.
(i) Find the critical numbers of f.
(ii) Find the absolute maximum and minimum of f on the interval $[-1, 2]$.

2. Let $f(x) = 4x^3 - 6x^2 + 1$.
(a) Find the intervals on which f is increasing or decreasing.
(b) Find the intervals of concavity and the inflection point(s).
(c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
(d) Sketch the graph of f.
(e) How many roots does f have?

3. A rectangular storage container with an open top is to have a volume of 20 m^3. The length of its base is twice the width. Material for the base costs \$5 per square meter. Material for the sides costs \$9 per square meter. Find the cost of materials for the cheapest such container. Indicate the corresponding dimensions.

4. A particle is moving along a straight line with the given data:

$$a(t) = 20t^3 - 30t^2 + 10t,\ s(0) = 0,\ s(1) = 0.$$

(i) Find the position function $s(t)$.
(ii) Find the velocity of the particle at $t = 0$.

5. Evaluate the integral.

(a) $\displaystyle\int_0^1 (1 + x^2)^2 dx$

(b) $\displaystyle\int_1^4 \frac{3y - 2}{\sqrt{y}} dy$

(c) $\displaystyle\int_0^{2\pi} \cos^3\theta d\theta$ (Hint: write $\cos^2\theta = 1 - \sin^2\theta$)

(d) $\displaystyle\int te^{-t^2} dt$ (Hint: recall that $\displaystyle\int e^u du = e^u + C$)

(e) $\displaystyle\int \frac{1}{s\ln s} ds$ (Hint: recall that $\displaystyle\int \frac{1}{u} du = \ln|u| + C$)