Math 231 Worksheet 8

1. Evaluate the integral by interpreting it in terms of areas.

$$(a)\ \int_{-1}^{1} 2|x|dx \qquad\qquad (b)\ \int_{0}^{2} \left(\sqrt{4-x^2}+2\right)dx$$

2. Evaluate the integral.

$$(a)\ \int_{0}^{1} 6x(1+x^2)dx \qquad\qquad (b)\ \int_{1}^{4} \frac{\sqrt{x}-4}{x^2}dx$$

Answers:

1. (a) The area represented by $\int_{-1}^{1} 2|x|dx$ consists of two right triangles above the x-axis with hight 2 and base 1. Therefore the integral is equal to $\boxed{2}$.

(b) Note that the graph of $y = \sqrt{4 - x^2}$ is the first quadrant of the circle centred at $(0, 0)$ and of radius 2. Therefore the graph of $\sqrt{4 - x^2} + 2$ is such a graph shifted up by 2, and the area represented by $\int_0^2 \big(\sqrt{4 - x^2} + 2\big)dx$ is equal to the area of the sector plus the area of a 2×2 square, that is $\boxed{\pi + 4}$.

2.

$$
\begin{aligned}
(a)\ \int_0^1 6x(1 + x^2)dx &= \int_0^1 6x + 6x^3 dx \\
&= \Big[6\frac{x^2}{2} + 6\frac{x^4}{4}\Big]_0^1 \\
&= \Big[3x^2 + 3\frac{x^4}{2}\Big]_0^1 \\
&= 3 + \frac{3}{2} \\
&= \boxed{\frac{9}{2}}
\end{aligned}
$$

$$
\begin{aligned}
(b)\ \int_1^4 \frac{\sqrt{x} - 4}{x^2}dx &= \int_1^4 \frac{\sqrt{x}}{x^2} - \frac{4}{x^2}dx \\
&= \int_1^4 x^{-3/2} - 4x^{-2}dx \\
&= \Big[-2x^{-1/2} + 4x^{-1}\Big]_1^4 \\
&= \Big[-\frac{2}{\sqrt{x}} + \frac{4}{x}\Big]_1^4 \\
&= (-1 + 1) - (-2 + 4) \\
&= \boxed{-2}
\end{aligned}
$$