6.1#2 The proof is the same as that of Lemma 6.1.5, with m_i replaced by M_i, and '$\le$' replaced by '$\ge$'. Note how to pass from $Q = P \cup \{c\}$ to general refinements of P.

6.1#6 Since $\underline{\int_a^b} f$ is defined as the sup of the lower sums, it suffices to show that

$$L(P, f) \ge 0$$

for any partition P. However, since $f \ge 0$, we have

$$L(P, f) = \sum_{k=1}^{n} m_k(f)\Delta x_k \ge 0$$

as each $m_k(f) \ge 0$.

6.1#8 Suppose $P = \{x_k\}_{k=1}^{n}$. Note that, since

$$f(x) + g(x) \le \sup_{[x_{k-1},x_k]} f + \sup_{[x_{k-1},x_k]} g$$

for all $x \in [x_{k-1}, x_k]$, we have

$$\sup_{[x_{k-1},x_k]} (f+g) \le \sup_{[x_{k-1},x_k]} f + \sup_{[x_{k-1},x_k]} g.$$

In other words,

$$M_k(f+g) \le M_k(f) + M_k(g).$$

Therefore

$$\begin{aligned} U(P, f+g) &= \sum_{k=1}^{n} M_k(f+g)\Delta x_k \\ &\le \sum_{k=1}^{n} \big(M_k(f) + M_k(g)\big)\Delta x_k \\ &= \sum_{k=1}^{n} M_k(f)\Delta x_k + \sum_{k=1}^{n} M_k(g)\Delta x_k \\ &= U(P, f) + U(P, g). \end{aligned}$$

This proves the inequality.