3.1#12 Given $\varepsilon > 0$, we need to find $M > a$ such that

$$|f(g(x)) - L| < \varepsilon, \ \forall x \geq M.$$

By the assumption that $\lim_{y\to\infty} f(y) = L$, there exists $K > a$ such that

$$|f(y) - L| < \varepsilon, \ \forall y \geq K.$$

Therefore if $y = g(x) \geq K$, we would have

$$|f(g(x)) - L| < \varepsilon.$$

On the other hand, by the assumption that $\lim_{x\to\infty} g(x) = \infty$, for this K there exists $M > a$ such that

$$g(x) \geq K, \ \forall x \geq M.$$

So, combining the above, we see that

$$|f(g(x)) - L| < \varepsilon$$

as long as $x \geq M$. This completes the proof.

3.2#4(a⇒b) To show that the sequence $\{f(x_n)\}$ converges to L, for any given $\varepsilon > 0$ we need to find n^* such that

$$|f(x_n) - L| < \varepsilon, \ \forall n \geq n^*.$$

Since $\lim_{x\to a} f(x) = L$, by definition for the $\varepsilon > 0$ above, there exists $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \text{ whenever } 0 < |x - a| < \delta, \ x \in D.$$

For this $\delta > 0$, since $\{x_n\}$ converges to a, there exists n_1^* such that

$$|x_n - a| < \delta, \ \forall n \geq n_1^*.$$

On the other hand, by assumption there exists n_2^* such that

$$x_n \neq a, \ \forall n \geq n_2^*.$$

If we let $n^* = \max(n_1^*, n_2^*)$, then it holds that

$$0 < |x_n - a| < \delta, \ x_n \in D, \ \forall n \geq n^*.$$

Thus for this n^*, we have

$$|f(x_n) - L| < \varepsilon, \ \forall n \geq n^*,$$

as desired.

3.3#3 Note that
$$f(x) = \frac{x}{x-1}$$
is well defined for $x > 1$. To show
$$\lim_{x \to 1^+} \frac{x}{x-1} = +\infty,$$
for any $M > 0$ we need to find $\delta > 0$ such that
$$\frac{x}{x-1} > M \text{ whenever } 0 < x - 1 < \delta.$$
Since $x > 1$ (i.e. $x - 1 > 0$), we have
$$\frac{x}{x-1} > \frac{1}{x-1}.$$
Thus it suffices to have
$$\frac{1}{x-1} > M,$$
or equivalently (again, using $x - 1 > 0$),
$$x - 1 < \frac{1}{M}.$$
If we choose $\delta = \frac{1}{M} > 0$, then provided $0 < x - 1 < \delta$, we have
$$\frac{x}{x-1} > M,$$
as desired.