5.1#3 (d) By the definition of $f(x)$, we have

$$f'(0) = \lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

The last limit equals 0 since $x \to 0$. Thus $f'(0) = 0$ (exists).
(e) By the definition of $f(x)$, we have

$$f'(0) = \lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

The last limit does not exists. Thus f is not differentiable at 0.

5.2#5 By Theorem 5.2.1(a), $f \pm g$ are both differentiable at $x = a$. By Theorem 5.2.3, since $F(y) = y^2$ is differentiable at all y, it follows that $(f \pm g)^2$ are both differentiable at $x = a$. By Theorem 5.2.1(a), it then follows that

$$fg = \frac{1}{4}(f+g)^2 - \frac{1}{4}(f-g)^2$$

is also differentiable at $x = a$; moreover

$$\begin{aligned}(fg)'(a) &= \frac{1}{2}(f(a)+g(a))(f'(a)+g'(a)) - \frac{1}{2}(f(a)-g(a))(f'(a)-g'(a)) \\ &= f'(a)g(a) + f(a)g'(a).\end{aligned}$$

A similar argument applies if one instead uses

$$fg = \frac{1}{2}(f+g)^2 - \frac{1}{2}f^2 - \frac{1}{2}g^2.$$