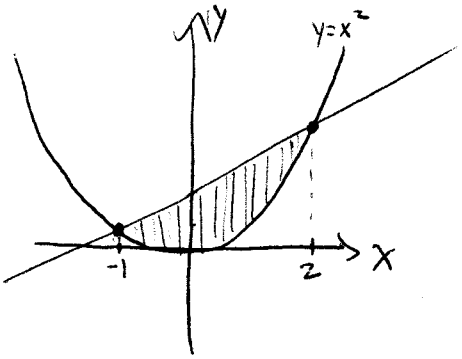
1. (10 pts) Find the area of the region enclosed by the given curves.

(a) $y = x^2,\ y = x + 2$



$x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = -1, 2.$

$$A = \int_{-1}^{2} \left[(x+2) - x^2\right] dx$$

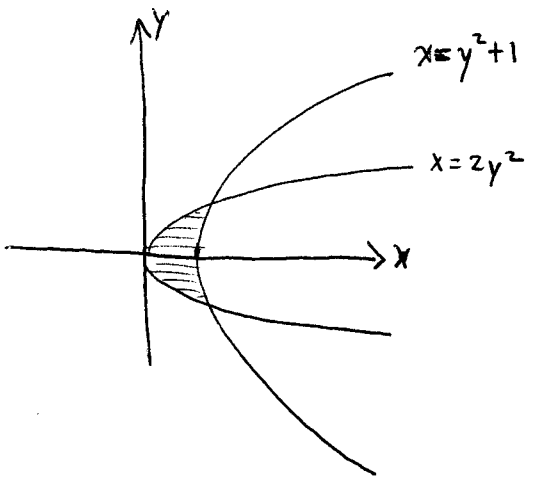
$$= \left[\frac{x^2}{2}\right]_{-1}^{2} + 2\cdot 3 - \left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \left[2 - \frac{1}{2}\right] + 6 - \left[\frac{8}{3} + \frac{1}{3}\right]$$

$$= \frac{3}{2} + 6 - 3$$

$$= \boxed{\frac{9}{2}}$$

(b) $x = 2y^2,\ x = y^2 + 1$ (note the direction of the parabolas)



$2y^2 = y^2 + 1$

$\Rightarrow y^2 = 1$

$\Rightarrow y = \pm 1$

$$A = \int_{-1}^{1} \left[(y^2+1) - 2y^2\right] dy$$

$$= \int_{-1}^{1} (1 - y^2)\, dy$$

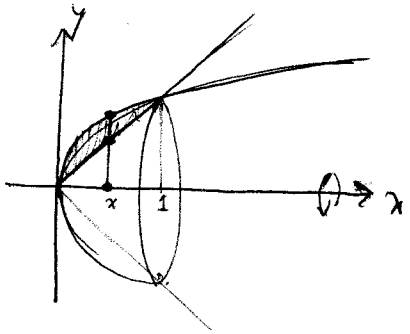
$$= 2 - \left[\frac{y^3}{3}\right]_{-1}^{1}$$

$$= 2 - \frac{2}{3}$$

$$= \boxed{\frac{4}{3}}$$

2. (10 pts) The region enclosed by the given curves is rotated about the specified axis. Find the volume of the resulting solid using cross-sections.

(a) $y = \sqrt{x}$, $y = x$; about the x-axis



$$V = \int_0^1 A(x)\,dx$$

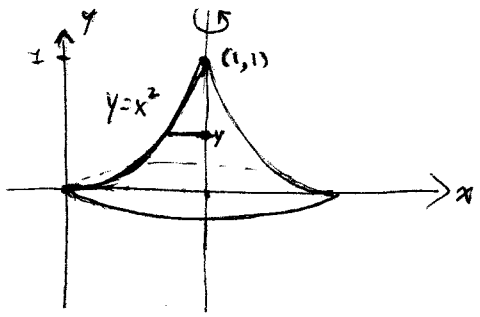
$$= \int_0^1 \pi(\sqrt{x})^2 - \pi(x)^2\,dx$$

$$= \pi \int_0^1 (x - x^2)\,dx$$

$$= \pi\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \boxed{\frac{\pi}{6}}$$

(b) $y = x^2$, $y = 0$, $x = 1$; about $x = 1$ (set up the integral only)



$$V = \int_0^1 A(y)\,dy$$

$$= \int_0^1 \pi(1 - \sqrt{y})^2\,dy$$

$$= \pi \int_0^1 (1 - 2\sqrt{y} + y)\,dy$$

$$= \pi\left(1 - \frac{4}{3} + \frac{1}{2}\right)$$

$$= \boxed{\frac{\pi}{6}}$$