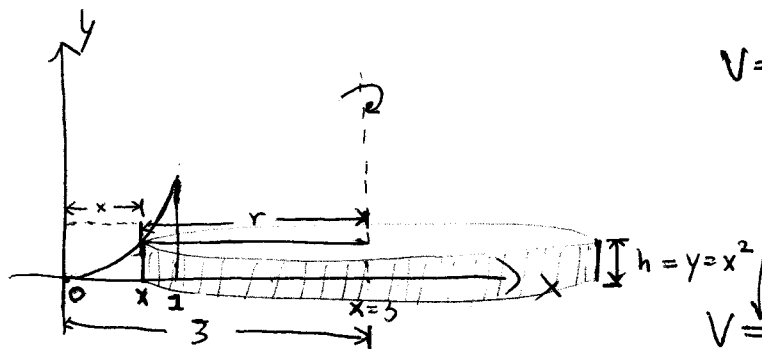


# Math 232 Quiz 2 Solutions

1. (a)

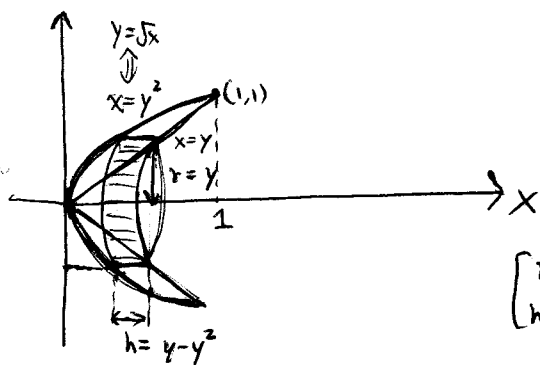


$$V = \int 2\pi r h dx$$

$$\begin{cases} r = 3-x \\ h = x^2 \end{cases} \quad (0 \leq x \leq 1)$$

$$V = \int_0^1 2\pi(3-x)x^2 dx = \boxed{\frac{3\pi}{2}}$$

1. (b)



$$V = \int 2\pi r h dy$$

$$\begin{cases} r = y \\ h = y - y^2 \end{cases} \Rightarrow \int_0^1 2\pi y(y - y^2) dy = \boxed{\frac{\pi}{6}}$$

2. (a)

$$L = \int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_0^3 \sqrt{1 + \left(\frac{x}{2}\right)^2} dx$$

$$= \int_0^3 \sqrt{1+x} dx$$

$$\begin{cases} u = 1+x \\ du = dx \end{cases} \Rightarrow \int_{u(0)}^{u(3)} \sqrt{u} du$$

$$= \int_1^4 u^{1/2} du$$

$$= \left[ \frac{2}{3} u^{3/2} \right]_1^4$$

$$= \boxed{\frac{14}{3}}$$

2. (b)  $\frac{dy}{dx} = \frac{\sqrt{x}}{2} - \frac{1}{2\sqrt{x}}$

$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{\sqrt{x}}{2}\right)^2 - 2 \cdot \frac{\sqrt{x}}{2} \cdot \frac{1}{2\sqrt{x}} + \left(\frac{1}{2\sqrt{x}}\right)^2$$

$$= \frac{x}{4} - \frac{1}{2} + \frac{1}{4x}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{x}{4} + \frac{1}{2} + \frac{1}{4x}$$

$$= \left(\frac{\sqrt{x}}{2}\right)^2 + 2 \cdot \frac{\sqrt{x}}{2} \cdot \frac{1}{2\sqrt{x}} + \left(\frac{1}{2\sqrt{x}}\right)^2$$

$$= \left(\frac{\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}\right)^2$$

$$\Rightarrow L = \int_1^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_1^4 \sqrt{\left(\frac{\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}\right)^2} dx$$

$$= \int_1^4 \left(\frac{\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}\right) dx$$

$$= \boxed{\frac{10}{3}}$$