Name: ____________________ Math 232 Quiz 3

1. (10 pts) Evaluate the integral using integration by parts.

(a) $\displaystyle\int_0^{\pi/2} \underbrace{x}_{u}\,\underbrace{\sin x\,dx}_{dv} = \left[uv\right]_0^{\pi/2} - \int_0^{\pi/2} v\,du$

$\begin{bmatrix} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{bmatrix}$

$$= \left[x(-\cos x)\right]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)\,dx$$

$$= -\frac{\pi}{2}\cancel{\cos\left(\frac{\pi}{2}\right)}^{0} + 0\cdot\cancel{\cos(0)}^{0} + \int_0^{\pi/2} \cos x\,dx$$

$$= \left[\sin x\right]_0^{\pi/2}$$

$$= \boxed{1}$$

(b) $\displaystyle\int \underbrace{x^2}_{u}\,\underbrace{e^x\,dx}_{dv} = uv - \int v\,du$

$\begin{bmatrix} u = x^2 & dv = e^x\,dx \\ du = 2x\,dx & v = e^x \end{bmatrix}$

$$= x^2 e^x - \int e^x\, 2x\,dx$$

$$= x^2 e^x - 2\int \underbrace{x}_{u}\,\underbrace{e^x dx}_{dv}$$

$$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2(xe^x - e^x) + C$$

$$= \boxed{e^x(x^2 - 2x + 2) + C}$$

2. (10 pts) Evaluate the trigonometric integral.

(a) $\displaystyle\int \cos^2 x\, dx$

$$= \int \frac{1+\cos(2x)}{2}\, dx$$

$$= \frac{1}{2}\int \left(1+\cos(2x)\right) dx$$

$$= \frac{1}{2}\left(x + \frac{\sin(2x)}{2}\right) + C$$

$$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin(2x) + C}$$

(b) $\displaystyle\int_0^{\pi/2} \sin^3 x\, dx$

$$= \int_0^{\pi/2} \sin^2 x \ \sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}} (1-\cos^2 x)\ d(-\cos x)$$

$$= -\int_0^{\pi/2} (1-\cos^2 x)\, d(\cos x)$$

[$u = \cos x$]

$$= -\int_1^0 (1-u^2)\, du$$

$$= \int_0^1 (1-u^2)\, du$$

$$= 1 - \frac{1}{3}$$

$$= \boxed{\frac{2}{3}}$$