Name: ______________________________ Math 232 Quiz 4

**1.** (20 pts) Evaluate the integral.

(6 pts) (a) $\displaystyle\int_0^{\pi/4} \tan^3 x\, dx$

$$= \int_0^{\pi/4} \tan x \tan^2 x \, dx$$

$$= \int_0^{\pi/4} \tan x \left(\sec^2 x - 1\right) dx$$

$$= \int_0^{\pi/4} \tan x \underbrace{\sec^2 x \, dx}_{d(\tan x)} - \int_0^{\pi/4} \tan x \, dx$$

$$= \left[\frac{\tan^2 x}{2}\right]_0^{\pi/4} - \Big[\ln|\sec x|\Big]_0^{\pi/4}$$

$$= \frac{1}{2} - \ln\sqrt{2} = \boxed{\frac{1}{2} - \frac{1}{2}\ln 2}$$

(7pts) (b) $\displaystyle\int x^3\sqrt{1-x^2}\, dx = \int \sin^3\theta \cos^2\theta \, d\theta$

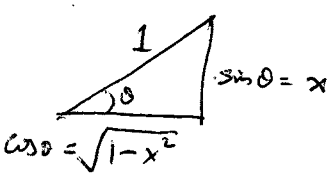
$\left[\begin{array}{l} x = \sin\theta \\ dx = \cos\theta \, d\theta \end{array}\right]$

$$= -\int \sin^2\theta \cos^2\theta \, d(\cos\theta)$$

$$= -\int (1-\cos^2\theta)(\cos^2\theta)\, d(\cos\theta)$$

$[u = \cos\theta]$

$$= -\int (1-u^2)u^2 \, du$$




$$= -\int (u^2 - u^4)\, du$$

$$= -\frac{u^3}{3} + \frac{u^5}{5} + C$$

$$= -\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5} + C$$

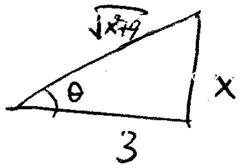
$$= \boxed{-\frac{\sqrt{1-x^2}^{\,3}}{3} + \frac{\sqrt{1-x^2}^{\,5}}{5} + C}$$

(7pts) (c) $\int \frac{1}{\sqrt{x^2+9}}\,dx = \int \frac{1}{\sqrt{9\tan^2\theta+9}}\,3\sec^2\theta\,d\theta$

$\tan\theta = \frac{x}{3} \Leftarrow \begin{bmatrix} x = 3\tan\theta \\ dx = 3\sec^2\theta\,d\theta \end{bmatrix}$

$$= \int \frac{1}{3\sec\theta} \cdot 3\sec^2\theta\,d\theta$$

$$= \int \sec\theta\,d\theta$$




$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

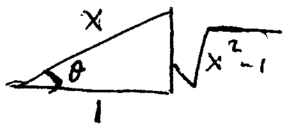
$$= \boxed{\ln\left|\sqrt{x^2+9} + x\right| + C}$$

(7pts) (d) $\int \frac{\sqrt{x^2-1}}{2x}\,dx = \int \frac{\sqrt{\sec^2\theta-1}}{2\sec\theta}\,\sec\theta\tan\theta\,d\theta$

$\theta = \operatorname{arcsec} x \Leftarrow \begin{bmatrix} x = \sec\theta \\ dx = \sec\theta\tan\theta\,d\theta \end{bmatrix}$

$$= \int \frac{\tan\theta}{2\cancel{\sec\theta}}\,\cancel{\sec\theta}\tan\theta\,d\theta$$

$$= \frac{1}{2}\int \tan^2\theta\,d\theta$$




$$= \frac{1}{2}\int (\sec^2\theta - 1)\,d\theta$$

$$= \frac{1}{2}(\tan\theta - \theta) + C$$

$$= \boxed{\frac{1}{2}\left(\sqrt{x^2-1} - \operatorname{arcsec} x\right) + C}$$