

# Math 232 Quiz 6 Solutions

#1. (a)  $\frac{dy}{dx} = xy^2$ ,  $y(0) = -2$ .

Solution.  $\frac{dy}{y^2} = x dx$

$$\int y^{-2} dy = \int x dx$$

$$-y^{-1} = \frac{x^2}{2} + C$$

$$\frac{1}{y} = -\frac{x^2}{2} + C \quad (C = -C)$$

$$y = \frac{1}{-\frac{x^2}{2} + C}$$

$$y(0) = \frac{1}{-0 + C} = -2 \Rightarrow C = -\frac{1}{2}$$

$$\Rightarrow y = \frac{1}{-\frac{x^2}{2} - \frac{1}{2}} = \boxed{\frac{-2}{x^2 + 1}}$$

(b)  $\frac{dy}{dx} = e^y$ ,  $y(0) = 0$

Solution.  $\frac{dy}{e^y} = dx$

$$\int e^{-y} dy = \int 1 dx$$

$$-e^{-y} = x + C$$

$$e^{-y} = -x + C \quad (C = -C)$$

$$\ln(e^{-y}) = \ln(-x + C)$$

$$-y = \ln(-x + C)$$

$$y = -\ln(-x + C)$$

$$y(0) = -\ln(-0 + C) = 0 \Rightarrow C = 1$$

$$\Rightarrow y = \boxed{-\ln(-x + 1)}$$

#2. (a)  $\lim_{n \rightarrow \infty} \frac{n^2 + n + 2}{n^2 + n} = \boxed{1}$

(b)  $\lim_{x \rightarrow \infty} \frac{n^2}{e^n} = \lim_{x \rightarrow \infty} \frac{x^2}{e^x} \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{2x}{e^x} \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{2}{e^x} = \boxed{0}$

(c)  $\lim_{n \rightarrow \infty} \cos\left(\frac{1}{2^n}\right) = \lim_{x \rightarrow 0} \cos(x) = \cos(0) = \boxed{1}$

(d)  $\lim_{n \rightarrow \infty} \frac{\sin(n^2)}{n} = 0 \quad \left( \frac{-1}{n} \leq \frac{\sin(n^2)}{n} \leq \frac{1}{n} \right)$