Name: ____________________ Math 232 Quiz 7

1. (5 pts each) Determine whether the series is convergent or divergent. *Find the sum* if it is convergent. If it is divergent, explain why.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n}$ $= \displaystyle\sum_{n=1}^{\infty} \frac{1}{3} \frac{2^{n-1}}{3^{n-1}} = \frac{1}{3} \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1}$

$= \frac{1}{3}\left(1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\right)$ (Geometric Series)

$= \frac{1}{3} \frac{1}{1-\left(\frac{2}{3}\right)}$ $\left(\displaystyle\sum_{n=1}^{\infty} r^{n-1} = \frac{1}{1-r}, \text{ if } |r|<1\right)$

$= \frac{1}{3} \frac{1}{\frac{1}{3}}$

$= \boxed{1}$

(convergent)

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2 + 2n + 1}$

Since $\displaystyle\lim_{n\to\infty} \frac{n^2}{n^2+2n+1} = 1$, we have that

$\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2+2n+1}$ DNE (in particular $\neq 0$).

By the Divergence Test, the series $\boxed{\text{diverges}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1+(-1)^n}{2^n} = \sum_{n=1}^{\infty}\left(\frac{1}{2^n} + \frac{(-1)^n}{2^n}\right)$

$$= \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n} \qquad \text{(sum law)}$$

$$= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty}\left(\frac{-1}{2}\right)^n$$

$$= \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} + \left(-\frac{1}{2}\right)\sum_{n=1}^{\infty}\left(\frac{-1}{2}\right)^{n-1} \qquad \text{(geometric series)}$$

$$= \frac{1}{2}\,\frac{1}{1-\left(\frac{1}{2}\right)} - \frac{1}{2}\,\frac{1}{1-\left(\frac{-1}{2}\right)}$$

$$= 1 - \frac{1}{2}\cdot\frac{2}{3}$$

$$= \boxed{\frac{2}{3}}$$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+2)} = \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right)$ (partial fractions)

$$= \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) + \left(\cancel{\frac{1}{5}} - \frac{1}{7}\right) + \cdots$$

$$= 1 + \frac{1}{2} \qquad \text{(one may justify using partial sums)}$$

$$= \boxed{\frac{3}{2}}$$