Name: ______________________ Math 232 Quiz 9

1. (6 pts) Find an equation of the tangent to the curve at the point corresponding to $t = 1$.

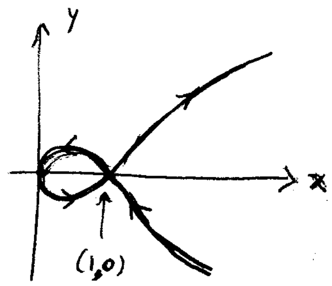
$$\begin{cases} x = t^2 \\ y = t^3 - t \end{cases}$$

@ $t=1$: $\begin{cases} m = \dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)} = \dfrac{3t^2-1}{2t} \overset{t=1}{=} \dfrac{3-1}{2} = 1 \\ (x_0, y_0) = (1, 0) \end{cases}$

$\Rightarrow$ $y - y_0 = m(x - x_0)$ (point-slope formula)

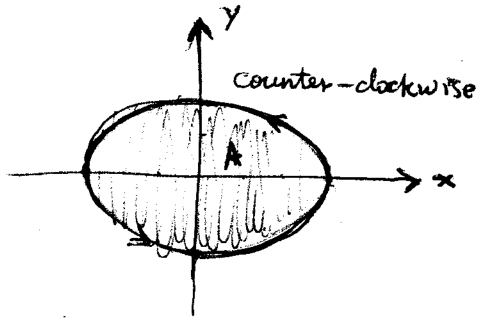
$y - 0 = 1(x-1)$

$\boxed{y = x - 1}$



2. (7 pts) Find the area enclosed by the curve (an ellipse)

$$\begin{cases} x = 3\cos\theta \\ y = 2\sin\theta \end{cases} \quad \text{where } 0 \le \theta \le 2\pi.$$



$A = \int_0^{2\pi} y\,dx$

$= \int_0^{2\pi} y(\theta)\,x'(\theta)\,d\theta$

$= \int_0^{2\pi} 2\sin\theta \cdot 3(-\sin\theta)\,d\theta$

$= -6\int_0^{2\pi} \sin^2\theta\,d\theta$

$= -6\int_0^{2\pi} \frac{1-\cos(2\theta)}{2}\,d\theta$

$= -3\int_0^{2\pi} [1 - \cos(2\theta)]\,d\theta$

$= -6\pi$ (net area)

$\Rightarrow |A| = \boxed{6\pi}$

3. (7 pts) Find the exact length of the curve

$$\begin{cases} x = 3t^2 \\ y = t^3 - 3t \end{cases} \qquad \text{where } 0 \le t \le 1.$$

$$L = \int_0^1 \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$= \int_0^1 \sqrt{(6t)^2 + (3t^2-3)^2}\, dt$$

$$= \int_0^1 \sqrt{36t^2 + (9t^4 - 18t^2 + 9)}\, dt$$

$$= \int_0^1 \sqrt{9t^4 + 18t^2 + 9}\, dt$$

$$= 3\int_0^1 \sqrt{(t^2+1)^2}\, dt$$

$$= 3\int_0^1 (t^2+1)\, dt$$

$$= 3\left(\frac{1}{3}+1\right)$$

$$= \boxed{4}$$

4. (6 pts) Find the Maclaurin series of the function.

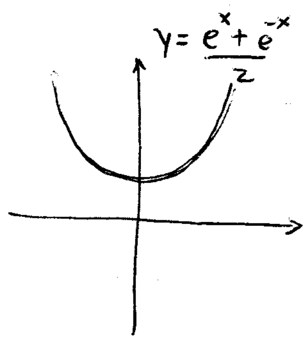
$$f(x) = \frac{1}{2}(e^x + e^{-x})$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$$e^{-x} = \sum_{n=0}^{\infty} \frac{1}{n!} u^n = \sum_{n=0}^{\infty} \frac{1}{n!}(-x)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$$

$u = -x$

$$= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \cdots$$



$$\Rightarrow e^x + e^{-x} = \sum_{n=0}^{\infty} \left(\frac{1}{n!} + \frac{(-1)^n}{n!}\right) x^n$$

$$= \sum_{n=0}^{\infty} \frac{2}{(2n)!} x^{2n}$$

$$\Rightarrow \frac{1}{2}(e^x + e^{-x}) = \frac{1}{2}\sum_{n=0}^{\infty} \frac{2}{(2n)!} x^{2n} = \boxed{\sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n}}$$

$$= \boxed{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots}$$