7.3#7(a) It can be shown that

$$\lim_{k\to\infty} \sqrt[k]{a_k} = \frac{2}{3} < 1.$$

Therefore, by the Root Test, the series converges.

7.3#7(b) It can be shown that

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{1}{4} < 1.$$

Therefore, by the Ratio Test, the series converges.

7.4#2(d) Note that

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{k}{4k^2-3}$$

is an alternating series. To show that it converges, by the Alternating Series Test, we need to show

$$\lim_{k\to\infty} \frac{k}{4k^2-3} = 0$$

(which is obvious) and that

$$\frac{k+1}{4(k+1)^2-3} \le \frac{k}{4k^2-3}.$$

The last inequality is equivalent to

$$(k+1)(4k^2-3) \le k\left(4(k+1)^2-3\right)$$

which is

$$4k^3+4k^2-3k-3 \le 4k^3+8k^2+k.$$

But this is clearly true since $k \ge 1$.

7.4#4 For any fixed n, by the triangle inequality, we have

$$\left|\sum_{k=1}^{n} a_k\right| \le \sum_{k=1}^{n} |a_k|.$$

Taking n to infinity, we obtain

$$\lim_{n\to\infty} \left|\sum_{k=1}^{n} a_k\right| \le \lim_{n\to\infty} \sum_{k=1}^{n} |a_k|,$$

or

$$\left|\lim_{n\to\infty} \sum_{k=1}^{n} a_k\right| \le \lim_{n\to\infty} \sum_{k=1}^{n} |a_k|.$$

By the definition of the value of series, this means

$$\left|\sum_{k=1}^{\infty} a_k\right| \le \sum_{k=1}^{\infty} |a_k|,$$

as desired.