8.6#1(c) When $|x| < 1$, we have $|x^2| < 1$, and therefore

$$\frac{1}{1-x^2} = \sum_{k=0}^{\infty} (x^2)^k = \sum_{k=0}^{\infty} x^{2k}.$$

So we can write, for $x \in (-1, 1)$,

$$\frac{x}{1-x^2} = x \cdot \sum_{k=0}^{\infty} x^{2k} = \sum_{k=0}^{\infty} x^{2k+1}.$$

8.6#1(g) Substituting $u = -x^2$ in the formula

$$e^u = \sum_{k=0}^{\infty} \frac{1}{k!} u^k,$$

we get

$$e^{-x^2} = \sum_{k=0}^{\infty} \frac{1}{k!} (-x^2)^k = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k},$$

which holds for all $x \in \mathbb{R}$.

8.6#2(b) Substituting $u = t^2$ in the formula

$$\sin(u) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} u^{2k+1},$$

we get

$$\sin(t^2) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} (t^2)^{2k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} t^{4k+2},$$

which holds for all $t \in \mathbb{R}$. Integrating from 0 to x, we get

$$\int_0^x \sin(t^2) dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \int_0^x t^{4k+2} dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \cdot \frac{x^{4k+3}}{4k+3},$$

which holds for all $x \in \mathbb{R}$.