**9.4#3(a)**

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \frac{t}{4}\left[\begin{pmatrix} 2 \\ -5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}\right] = \begin{pmatrix} \frac{3}{4}t - 1 \\ -2t + 3 \end{pmatrix}.$$

**9.4#3(b)** For instance:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} -2\sin(t) \\ 2\cos(t) \end{pmatrix}.$$

**9.6#9** Suppose

$$\vec{r}(t) = \begin{pmatrix} f(t) \\ g(t) \\ h(t) \end{pmatrix}.$$

Since $\vec{r}(t)$ is continuous, the functions $f, g, h$ are all continuous. In particular, the nonnegative function

$$[f(t)]^2 + [g(t)]^2 + [h(t)]^2$$

is continuous. Writing

$$\|\vec{r}(t)\| = \sqrt{[f(t)]^2 + [g(t)]^2 + [h(t)]^2},$$

we see that, as the square root of a nonnegative continuous function, $\|\vec{r}(t)\|$ is also continuous.

However, the continuity of $\|\vec{r}(t)\|$ does not in general imply the continuity of $\vec{r}(t)$. For instance, if we let

$$\vec{r}(t) = \begin{pmatrix} f(t) \\ 0 \\ 0 \end{pmatrix}, \quad t \in \mathbb{R}$$

where

$$f(t) = \begin{cases} -1 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}.$$

Then

$$\|\vec{r}(t)\| = 1, \ \forall t \in \mathbb{R}$$

and therefore $\|\vec{r}(t)\|$ is continuous on $\mathbb{R}$. However the vector-valued function $\vec{r}(t)$ is not continuous on $\mathbb{R}$ as $f(t)$ is discontinuous at $t = 0$.

**9.6#11(c)** Suppose

$$\vec{u}(t) = \begin{pmatrix} f(t) \\ g(t) \\ h(t) \end{pmatrix}.$$

Then

$$\begin{aligned}
\frac{d}{dt}[k(t)\vec{u}(t)] = \frac{d}{dt}\begin{pmatrix} k(t)f(t) \\ k(t)g(t) \\ k(t)h(t) \end{pmatrix} &= \begin{pmatrix} k'(t)f(t) + k(t)f'(t) \\ k'(t)g(t) + k(t)g'(t) \\ k'(t)h(t) + k(t)h'(t) \end{pmatrix} \\
&= \begin{pmatrix} k'(t)f(t) \\ k'(t)g(t) \\ k'(t)h(t) \end{pmatrix} + \begin{pmatrix} k(t)f'(t) \\ k(t)g'(t) \\ k(t)h'(t) \end{pmatrix} \\
&= k'(t)\vec{u}(t) + k(t)\vec{u}'(t).
\end{aligned}$$

This proves the identity.

**9.6#11(d)** Suppose

$$\vec{u}(t) = \begin{pmatrix} f(t) \\ g(t) \\ h(t) \end{pmatrix}.$$

Then

$$\begin{aligned}
\frac{d}{dt}\vec{u}(k(t)) = \frac{d}{dt}\begin{pmatrix} f(k(t)) \\ g(k(t)) \\ h(k(t)) \end{pmatrix} &= \begin{pmatrix} f(k(t))k'(t) \\ g(k(t))k'(t) \\ h(k(t))k'(t) \end{pmatrix} \\
&= k'(t)\begin{pmatrix} f(k(t)) \\ g(k(t)) \\ h(k(t)) \end{pmatrix} \\
&= k'(t)\vec{u}(k(t)).
\end{aligned}$$

This proves the identity.