9.6#27 Suppose

$$\vec{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \end{bmatrix}, \quad \vec{R}(t) = \begin{bmatrix} R_1(t) \\ R_2(t) \\ R_3(t) \end{bmatrix}.$$

By assumption we have

$$\vec{R}\,'(t) = \begin{bmatrix} R_1'(t) \\ R_2'(t) \\ R_3'(t) \end{bmatrix} = \begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \end{bmatrix} = \vec{r}\,'(t).$$

Therefore, by the FTC,

$$\begin{aligned}
\int_a^b \vec{r}(t)\,dt &= \int_a^b \vec{R}\,'(t)\,dt \\
&= \begin{bmatrix} \int_a^b R_1'(t)dt \\ \int_a^b R_2'(t)dt \\ \int_a^b R_3'(t)dt \end{bmatrix} \\
&= \begin{bmatrix} R_1(b) - R_1(a) \\ R_2(b) - R_2(a) \\ R_3(b) - R_3(a) \end{bmatrix} \\
&= \begin{bmatrix} R_1(b) \\ R_2(b) \\ R_3(b) \end{bmatrix} - \begin{bmatrix} R_1(a) \\ R_2(a) \\ R_3(a) \end{bmatrix} \\
&= \vec{R}(b) - \vec{R}(a).
\end{aligned}$$

This proves the identity.

9.6#28 (a) Suppose

$$\vec{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \quad \vec{v}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}.$$

Then

$$
\begin{aligned}
\int_a^b [\vec{u}(t) + \vec{v}(t)]\, dt &= \int_a^b \begin{bmatrix} u_1(t) + v_1(t) \\ u_2(t) + u_v(t) \\ u_3(t) + v_3(t) \end{bmatrix} dt \\
&= \begin{bmatrix} \int_a^b (u_1(t) + v_1(t))dt \\ \int_a^b (u_2(t) + v_2(t))dt \\ \int_a^b (u_3(t) + v_3(t))dt \end{bmatrix} \\
&= \begin{bmatrix} \int_a^b u_1(t)dt + \int_a^b v_1(t)dt \\ \int_a^b u_2(t)dt + \int_a^b v_2(t)dt \\ \int_a^b u_3(t)dt + \int_a^b v_3(t)dt \end{bmatrix} \\
&= \begin{bmatrix} \int_a^b u_1(t)dt \\ \int_a^b u_2(t)dt \\ \int_a^b u_3(t)dt \end{bmatrix} + \begin{bmatrix} \int_a^b v_1(t)dt \\ \int_a^b v_2(t)dt \\ \int_a^b v_3(t)dt \end{bmatrix} \\
&= \int_a^b \vec{u}(t)\, dt + \int_a^b \vec{v}(t)\, dt.
\end{aligned}
$$

This proves the identity.
(b) Suppose $c \in \mathbb{R}$ is a constant. Then

$$
\begin{aligned}
\int_a^b c\, \vec{u}(t)\, dt &= \int_a^b \begin{bmatrix} c\, u_1(t)dt \\ c\, u_2(t)dt \\ c\, u_3(t)dt \end{bmatrix} dt \\
&= \begin{bmatrix} \int_a^b c\, u_1(t)dt \\ \int_a^b c\, u_2(t)dt \\ \int_a^b c\, u_3(t)dt \end{bmatrix} \\
&= \begin{bmatrix} c \int_a^b u_1(t)dt \\ c \int_a^b u_2(t)dt \\ c \int_a^b u_3(t)dt \end{bmatrix} \\
&= c \begin{bmatrix} \int_a^b u_1(t)dt \\ \int_a^b u_2(t)dt \\ \int_a^b u_3(t)dt \end{bmatrix} \\
&= c \int_a^b \vec{u}(t)\, dt.
\end{aligned}
$$

This proves the identity.

9.6#29 Suppose

$$
\vec{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \quad \vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}.
$$

Then

$$\begin{aligned}
\int_a^b \vec{c}\cdot\vec{u}(t)\,dt &= \int_a^b \big(c_1u_1(t)+c_2u_2(t)+c_3u_3(t)\big)\,dt \\
&= c_1\int_a^b u_1(t)dt + c_2\int_a^b u_2(t)dt + c_3\int_a^b u_3(t)dt \\
&= \vec{c}\cdot\begin{bmatrix}\int_a^b u_1(t)dt\\ \int_a^b u_2(t)dt\\ \int_a^b u_3(t)dt\end{bmatrix} \\
&= \vec{c}\cdot\int_a^b \vec{u}(t)\,dt.
\end{aligned}$$

This proves the identity.

9.7#11 (a,b) Since

$$\gamma'(u) = \begin{bmatrix}-\sin(u^2)2u\\ \cos(u^2)2u\end{bmatrix} = 2u\begin{bmatrix}-\sin(u^2)\\ \cos(u^2)\end{bmatrix},$$

we have

$$\|\gamma'(u)\| = 2u.$$

This shows the speed of the particle traveling along the curve is given by $2u$. By the arclength formula, we then have

$$s(t) = \int_0^t \|\gamma'(u)\|du = \int_0^t 2u\,du = t^2.$$

(c) From the arclength formula $s = t^2$ above, we have $t = \sqrt{s}$. Substituting, we obtain

$$\gamma(t) = \gamma(\sqrt{s}) = \begin{bmatrix}\cos(s)\\ \sin(s)\end{bmatrix}.$$

In particular, we have

$$\left\|\frac{d}{ds}\gamma\right\| = 1.$$

Therefore the speed of the particle traveling according the (arclength) parameter s is 1.