7.2#1(k) Notice that

$$\ln k \geq 1, \ \forall k \geq 3.$$

Therefore,

$$\frac{\ln k}{k} \geq \frac{1}{k}, \ \forall k \geq 3.$$

Since

$$\sum_{k=3}^{\infty} \frac{1}{k}$$

diverges. By the Comparison Test, we see that

$$\sum_{k=3}^{\infty} \frac{\ln k}{k}$$

diverges as well.

7.2#1(s) Consider

$$b_k = \frac{\sqrt{k}}{\sqrt{k^3}} = \frac{1}{k}.$$

Then

$$\lim_{k\to\infty} \frac{b_k}{a_k} = \frac{\sqrt{k^3}+1}{\sqrt{k^3}} = 1 < \infty.$$

Since $\sum b_k$ diverges, by the Limit Comparison Test, we see that

$$\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k^3}+1}$$

diverges as well.

7.2#5 Since $\sum_{k=1}^{\infty} a_k$ converges, we must have

$$\lim_{k\to\infty} a_k = 0.$$

This in particular implies that $\{a_k\}$ is bounded, i.e. there exists $M > 0$ such that

$$a_k = |a_k| \leq M, \ \forall k \geq 1.$$

From this we deduce that

$$0 \leq a_k^2 = a_k a_k \leq M a_k, \ \forall k \geq 1.$$

Since $\sum M a_k$ converges, by the Comparison Test, we see that $\sum_{k=1}^{\infty} a_k^2$ converges too. This completes the proof.

7.3#7(a) It can be shown that

$$\lim_{k\to\infty} \sqrt[k]{a_k} = \frac{2}{3} < 1.$$

Therefore, by the Root Test, the series converges.

7.3#7(b) It can be shown that

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{1}{4} < 1.$$

Therefore, by the Ratio Test, the series converges.