8.4#1(a) Notice that, for $k \geq 1$,

$$\left|\frac{x^k}{k^2}\right| \leq \frac{1}{k^2}, \ \forall x \in [0,1].$$

Since $\sum_{k=1}^{\infty} \frac{1}{k^2} < \infty$, by the M-Test, the series

$$\sum_{k=1}^{\infty} \frac{x^k}{k^2}$$

converges uniformly on $[0, 1]$.

8.4#1(j) Notice that, for $k \geq 1$,

$$\left|x^k\right| \leq a^k, \ \forall x \in [-a,a].$$

Since $a < 1$, we have $\sum_{k=1}^{\infty} a^k < \infty$. Therefore, by the M-Test, the series

$$\sum_{k=1}^{\infty} x^k$$

converges uniformly on $[-a, a]$, i.e. for x with $|x| \leq a$.

8.5#5 Pick a number s with $r < s < R$. Since $x_0 = a + s$ is (strictly) within the radius of convergence, we have that the series

$$\sum_{k=1}^{\infty} a_k(x_0 - a)^k = \sum_{k=1}^{\infty} a_k s^k$$

converges. In particular, this implies

$$\lim_{k\to\infty} a_k s^k = 0$$

and therefore the sequence $\{a_k s^k\}_{k=1}^{\infty}$ is bounded, i.e. there exists $M > 0$ such that

$$|a_k s^k| \leq M, \ \forall k \geq 1.$$

Now for any $x \in [a - r, a + r]$, we have that

$$|a_k(x-a)^k| \leq |a_k| r^k = |a_k| s^k \cdot \frac{r^k}{s^k} \leq M \left(\frac{r}{s}\right)^k.$$

Since $r < s$ (and therefore $\frac{r}{s} < 1$), the series

$$\sum_{k=1}^{\infty} M \left(\frac{r}{s}\right)^k < \infty.$$

By the M-Test, this implies that

$$\sum_{k=1}^{\infty} a_k(x-a)^k$$

converges uniformly on $[a - r, a + r]$, as desired.