9.6#9 Suppose

$$\vec{r}(t) = \begin{pmatrix} f(t) \\ g(t) \\ h(t) \end{pmatrix}.$$

Since $\vec{r}(t)$ is continuous, the functions f, g, h are all continuous. In particular, the nonnegative function

$$[f(t)]^2 + [g(t)]^2 + [h(t)]^2$$

is continuous. Writing

$$\|\vec{r}(t)\| = \sqrt{[f(t)]^2 + [g(t)]^2 + [h(t)]^2},$$

we see that, as the square root of a nonnegative continuous function, $\|\vec{r}(t)\|$ is also continuous.

However, the continuity of $\|\vec{r}(t)\|$ does not in general imply the continuity of $\vec{r}(t)$. For instance, if we let

$$\vec{r}(t) = \begin{pmatrix} f(t) \\ 0 \\ 0 \end{pmatrix}, \quad t \in \mathbb{R}$$

where

$$f(t) = \begin{cases} -1 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}.$$

Then

$$\|\vec{r}(t)\| = 1, \ \forall t \in \mathbb{R}$$

and therefore $\|\vec{r}(t)\|$ is continuous on $\mathbb{R}$. However the vector-valued function $\vec{r}(t)$ is not continuous on $\mathbb{R}$ as $f(t)$ is discontinuous at $t = 0$.

9.6#11(c) Suppose

$$\vec{u}(t) = \begin{pmatrix} f(t) \\ g(t) \\ h(t) \end{pmatrix}.$$

Then

$$\begin{aligned} \frac{d}{dt}[k(t)\vec{u}(t)] = \frac{d}{dt}\begin{pmatrix} k(t)f(t) \\ k(t)g(t) \\ k(t)h(t) \end{pmatrix} &= \begin{pmatrix} k'(t)f(t) + k(t)f'(t) \\ k'(t)g(t) + k(t)g'(t) \\ k'(t)h(t) + k(t)h'(t) \end{pmatrix} \\ &= \begin{pmatrix} k'(t)f(t) \\ k'(t)g(t) \\ k'(t)h(t) \end{pmatrix} + \begin{pmatrix} k(t)f'(t) \\ k(t)g'(t) \\ k(t)h'(t) \end{pmatrix} \\ &= k'(t)\vec{u}(t) + k(t)\vec{u}'(t). \end{aligned}$$

This proves the identity.

9.6#11(d) Suppose

$$\vec{u}(t) = \begin{pmatrix} f(t) \\ g(t) \\ h(t) \end{pmatrix}.$$

Then

$$\frac{d}{dt}\vec{u}(k(t)) = \frac{d}{dt}\begin{pmatrix} f(k(t)) \\ g(k(t)) \\ h(k(t)) \end{pmatrix} = \begin{pmatrix} f(k(t))k'(t) \\ g(k(t))k'(t) \\ h(k(t))k'(t) \end{pmatrix}$$
$$= k'(t)\begin{pmatrix} f(k(t)) \\ g(k(t)) \\ h(k(t)) \end{pmatrix}$$
$$= k'(t)\vec{u}(k(t)).$$

This proves the identity.

9.6#29 Suppose

$$\vec{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \quad \vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}.$$

Then

$$\int_a^b \vec{c}\cdot\vec{u}(t)\,dt = \int_a^b \big(c_1u_1(t) + c_2u_2(t) + c_3u_3(t)\big)\,dt$$
$$= c_1\int_a^b u_1(t)dt + c_2\int_a^b u_2(t)dt + c_3\int_a^b u_3(t)dt$$
$$= \vec{c}\cdot\begin{bmatrix} \int_a^b u_1(t)dt \\ \int_a^b u_2(t)dt \\ \int_a^b u_3(t)dt \end{bmatrix}$$
$$= \vec{c}\cdot\int_a^b \vec{u}(t)\,dt.$$

This proves the identity.

9.7#11 (a,b) Since

$$\gamma'(u) = \begin{bmatrix} -\sin(u^2)2u \\ \cos(u^2)2u \end{bmatrix} = 2u\begin{bmatrix} -\sin(u^2) \\ \cos(u^2) \end{bmatrix},$$

we have

$$\|\gamma'(u)\| = 2u.$$

This shows the speed of the particle traveling along the curve is given by $2u$. By the arclength formula, we then have

$$s(t) = \int_0^t \|\gamma'(u)\|du = \int_0^t 2u\,du = t^2.$$

(c) From the arclength formula $s = t^2$ above, we have $t = \sqrt{s}$. Substituting, we obtain

$$\gamma(t) = \gamma(\sqrt{s}) = \begin{bmatrix} \cos(s) \\ \sin(s) \end{bmatrix}.$$

In particular, we have

$$\left\| \frac{d}{ds}\gamma \right\| = 1.$$

Therefore the speed of the particle traveling according the (arclength) parameter s is 1.