Name: **Math 1441 Practice Midterm 1**

You have 75 minutes to finish the exam. This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

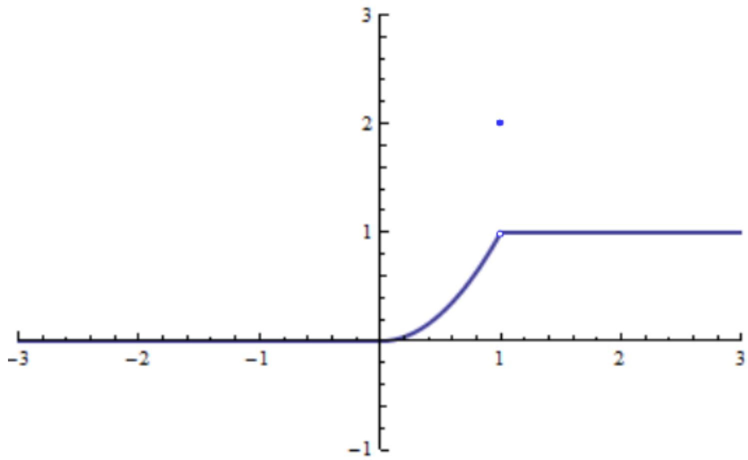
Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. Let

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x^2 & \text{if } 0 < x < 1 \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x > 1. \end{cases}$$

(a) (2 points) Sketch the graph of f.



(b) (3 points) Determine the values of c at which $\lim_{x\to c} f(x)$ exists.

All real numbers.

(c) (3 points) Determine the values of c at which f is continuous.

All real numbers except $c = 1$.

(d) (2 points) Find the horizontal asymptote(s).

$y = 0$ *and* $y = 1$.

2. Find the limit, if it exists.

(a) (3 points)

$$\lim_{x \to -2} \frac{x^2+1}{x+1}$$

<u>Solution</u>. This is a rational function, therefore is continuous on its domain. Since -2 is in the domain, we can do direct substitution with $x = -2$ to get

$$\lim_{x \to -2} \frac{x^2+1}{x+1} = \frac{(-2)^2+1}{(-2)+1} = \boxed{-5}.$$

(b) (3 points)

$$\lim_{x \to 3} (2x+1)(x-1)^2$$

<u>Solution</u>. This is a polynomial (which can be seen by multiplying out the product), therefore is continuous on the real line. In particular, we can do direct substitution with $x = 3$ to get

$$\lim_{x \to 3} (2x+1)(x-1)^2 = (2 \cdot 3 + 1)(3-1)^2 = \boxed{28}.$$

(c) (4 points)

$$\lim_{x \to 1} \frac{x}{\sqrt{4-x^2}}$$

<u>Solution</u>. Applying the Distributive Law to the quotient, and then to the square root, we get

$$\lim_{x \to 1} \frac{x}{\sqrt{4-x^2}} = \frac{\lim_{x \to 1} x}{\sqrt{\lim_{x \to 1}(4-x^2)}} = \frac{1}{\sqrt{4-1^2}} = \boxed{\frac{1}{\sqrt{3}}}.$$

3. Evaluate the limit.

(a) (5 points)

$$\lim_{x\to 10} \frac{9x^2-900}{x^2-11x+10}$$

Solution.

$$\begin{aligned}
\lim_{x\to 10} \frac{9x^2-900}{x^2-11x+10} &= \lim_{x\to 10} \frac{9(x^2-100)}{x^2-11x+10} && \textit{(factor out 9)} \\
&= \lim_{x\to 10} \frac{9(x^2-10^2)}{(x-10)(x-1)} && \textit{(factor the denominator)} \\
&= \lim_{x\to 10} \frac{9\cancel{(x-10)}(x+10)}{\cancel{(x-10)}(x-1)} && \textit{(factor the numinator)} \\
&= \lim_{x\to 10} \frac{9(x+10)}{x-1} && \textit{(simplify)} \\
&= \frac{9(10+10)}{10-1} && \textit{(direct substitution)} \\
&= \boxed{20}
\end{aligned}$$

(b) (5 points)

$$\lim_{x\to -\infty} \frac{6x^4+3x^2+2x}{2x^4+2x^3-x}$$

Solution.

$$\begin{aligned}
\lim_{x\to -\infty} \frac{6x^4+3x^2+2x}{2x^4+2x^3-x} &= \lim_{x\to -\infty} \frac{\frac{6x^4}{x^4}+\frac{3x^2}{x^4}+\frac{2x}{x^4}}{\frac{2x^4}{x^4}+\frac{2x^3}{x^4}-\frac{x}{x^4}} && \textit{(divide through by } x^4\textit{)} \\
&= \lim_{x\to -\infty} \frac{6+\frac{3}{x^2}+\frac{2}{x^3}}{2+\frac{2}{x}-\frac{1}{x^3}} && \textit{(simplify)} \\
&= \frac{6+\frac{3}{(-\infty)^2}+\frac{2}{(-\infty)^3}}{2+\frac{2}{(-\infty)}-\frac{1}{(-\infty)^3}} && \textit{(direct substitution)} \\
&= \frac{6+\frac{3}{\infty}+\frac{2}{-\infty}}{2+\frac{2}{-\infty}-\frac{1}{-\infty}} \\
&= \frac{6+0+0}{2+0-0} \\
&= \boxed{3}
\end{aligned}$$

4. Find the limit, if it exists. Otherwise explain why the limit does not exist.

(a) (3 points)

$$\lim_{x\to 2^-} \frac{x-3}{x-2}$$

Solution.

$$\begin{aligned}
\lim_{x\to 2^-} \frac{x-3}{x-2} &= \frac{2^- - 3}{2^- - 2} && (\textit{direct substitution}) \\
&= \frac{-1}{0^-} && (\textit{using } c^- - c = 0^-) \\
&= -\frac{1}{0^-} \\
&= -(-\infty) && \left(\textit{using } \frac{1}{0^-} = -\infty\right) \\
&= \boxed{\infty}
\end{aligned}$$

(b) (3 points)

$$\lim_{x\to 0} \frac{1}{x^3}$$

Solution. Direct substitution gives $\frac{1}{0}$ (indeterminate). So we need to examine the one-sided limits. For the right-hand limit, we have

$$\lim_{x\to 0^+} \frac{1}{x^3} = \frac{1}{(0^+)^3} = \frac{1}{0^+} = \infty;$$

for the left-hand limit, we have

$$\lim_{x\to 0^-} \frac{1}{x^3} = \frac{1}{(0^-)^3} = \frac{1}{0^-} = -\infty.$$

Since the left/right-hand limits disagree, the two-sided limit $\boxed{DNE}$.

(c) (4 points)

$$\lim_{x\to 3} \frac{1}{(x-3)^2}$$

Solution. Again, direct substitution gives $\frac{1}{0}$. For the one-sided limits, we have

$$\lim_{x\to 3^+} \frac{1}{(x-3)^2} = \frac{1}{(3^+-3)^2} = \frac{1}{(0^+)^2} = \frac{1}{0^+} = \infty,$$

$$\lim_{x\to 3^-} \frac{1}{(x-3)^2} = \frac{1}{(3^--3)^2} = \frac{1}{(0^-)^2} = \frac{1}{0^+} = \infty.$$

Since the left/right-hand limits are both equal to ∞, the two-sided limit

$$\lim_{x\to 3} \frac{1}{(x-3)^2} = \boxed{\infty}.$$

5. (10 points) Find the horizontal and vertical asymptotes. Justify your answers.

$$y = \frac{x-1}{x^2-1}$$

Solution. *(i) To find the horizontal asymptote(s), we need to find*

$$\lim_{x\to\infty} \frac{x-1}{x^2-1} \quad \textit{and} \quad \lim_{x\to-\infty} \frac{x-1}{x^2-1}.$$

For the limit at ∞, we have

$$\begin{aligned}
\lim_{x\to\infty} \frac{x-1}{x^2-1} &= \lim_{x\to\infty} \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} && (\textit{divide through by } x^2) \\
&= \lim_{x\to\infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 - \frac{1}{x^2}} && (\textit{simplify}) \\
&= \frac{\frac{1}{\infty} - \frac{1}{\infty^2}}{1 - \frac{1}{\infty^2}} && (\textit{direct substitution}) \\
&= \frac{0-0}{1-0} \\
&= \boxed{0}.
\end{aligned}$$

Since $\frac{x-1}{x^2-1}$ is a rational function, the limit at $-\infty$ must equal 0 as well. Therefore there is only one horizontal asymptote: $\boxed{y=0}$.

(ii) To find the vertical asymptote(s), it is useful to notice that

$$\frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1}.$$

The function $\frac{1}{x+1}$ is continuous except at $x=-1$ (where it is undefined). To see if $x=-1$ is a vertical asymptote, we need to examine

$$\lim_{x\to-1^+} \frac{1}{x+1} \quad \textit{and/or} \quad \lim_{x\to-1^-} \frac{1}{x+1}.$$

For the right-hand limit, we have

$$\lim_{x\to-1^+} \frac{1}{x+1} = \frac{1}{-1^+ + 1} = \frac{1}{(-1)^+ - (-1)} = \frac{1}{0^+} = \infty.$$

Thus $\boxed{x=-1}$ *is a indeed vertical asymptote, and is the only vertical asymptote.*