In this note we present a recent result of Körner [3] on Salem sets. Throughout, we shall fix $0 < \alpha < 1$ and $C = (2\pi)^{1-\alpha}$.

We first fix some notations. Let $\mathbb{T}$ be the unit circle in the plane equipped with the standard measure τ with $\tau(\mathbb{T}) = 2\pi$. A subset $I \subset \mathbb{T}$ is called an interval if it is connected, and $\tau(I)$ is denote by $|I|$. If μ be a finite Borel measure on $\mathbb{T}$, the Fourier transform of μ is defined by

$$\hat{\mu}(r) = \int_{\mathbb{T}} e^{-irt} d\mu(t)$$

where $r \in \mathbb{Z}$. Here we have identified $\mathbb{T}$ with the interval $[-\pi, \pi)$ in the usual way. Note that $\hat{\mu}(0) = \mu(\mathbb{T})$ and that $\hat{\tau}(r) = 0$ whenever $r \neq 0$. We say that μ is supported on a Borel set $E \subset \mathbb{T}$ if $|\mu|(\mathbb{T} \backslash E) = 0$. Finally, we equip $\mathbb{T}$ with the intrinsic metric, denoted by $|x - y|$ for $x, y \in \mathbb{T}$.

Now consider the following metric spaces. The last one will be the space in which we shall run the Baire category argument.

Lemma 1. *(i) Let $\mathcal{F}$ be the space consisting of non-empty closed subset of $\mathbb{T}$, equipped with the Hausdorff distance, i.e.*

$$d_{\mathcal{F}}(E, F) = \sup_{e \in E} \inf_{f \in F} |e - f| + \sup_{f \in F} \inf_{e \in E} |f - e|.$$

Then $(\mathcal{F}, d_{\mathcal{F}})$ a complete (in fact compact) metric space.

(ii) Let $\mathcal{E}$ be the space consisting of ordered pairs (E, μ) where $E \in \mathcal{F}$ and μ is a finite nonnegative Borel measure supported on E such that

$$\lim_{|r| \to \infty} |r|^{\alpha/2} \hat{\mu}(r) = 0. \tag{1}$$

Define

$$d_{\mathcal{E}}\big((E, \mu), (F, \sigma)\big) = d_{\mathcal{F}}(E, F) + |\hat{\mu}(0) - \hat{\sigma}(0)| + \sup_{r \in \mathbb{Z}} |r|^{\alpha/2} |\hat{\mu}(r) - \hat{\sigma}(r)|.$$

Then $(\mathcal{E}, d_{\mathcal{E}})$ is a non-empty complete metric space.

(iii) Let $\mathcal{G}$ be the subspace of $\mathcal{E}$ consisting of pairs (E, μ) such that

$$\mu(I) \leq C|I|^{\alpha} \tag{2}$$

for all interval I. Then $\mathcal{G}$ is a non-empty closed subspace of $\mathcal{E}$. In particular, $(\mathcal{G}, d_{\mathcal{G}})$ is a non-empty complete metric space, where $d_{\mathcal{G}} := d_{\mathcal{E}}$.

Proof. (i) See e.g.

http://www-math.mit.edu/phase2/UJM/vol1/HAUSF.PDF.

(ii) $d_{\mathcal{E}}$ is nondegenerate because $\hat{\mu}$ uniquely determines μ. Let (E_k, μ_k) be a Cauchy sequence in $\mathcal{E}$, then $\mu_k(\mathbb{T}) = \hat{\mu}_k(0)$ is bounded and hence $\|\mu_k\|$ is bounded. Using Riesz representation theorem and Banach-Alaoglu theorem, one can extract a subsequence converging weakly to a finite measure μ. To see that μ is nonnegative one tests with bump functions and uses the (automatic) outer or inner regularity of μ. To see that μ is supported on the limiting set E one again applies test functions to make use of the Hausdorff convergence. The rest of the verification is essentially the completeness of sequence space c_0 applied to the sequence $\{|r|^{\alpha/2}\hat{\mu}(r)\}$.

(iii) $\mathcal{G}$ is non-empty because it contains $(\mathbb{T}, \tau)$. The proof for closedness is similar to the argument in (ii). □

As usual in Baire category argument, we now turn to define dense open (to be shown) subsets of $(\mathcal{G}, d_{\mathcal{G}})$.

Definition 1. Suppose $\alpha < \gamma < 1$ and $\epsilon > 0$. Define $\mathcal{A}_{\gamma,\epsilon}$ to be the the subset of $\mathcal{G}$ consisting of pairs (E, μ) such that we can find intervals $I_1, \cdots, I_M$ with

$$E \subset \bigcup_{m=1}^{M} I_m$$

and $|I_1| = \cdots = |I_M| < \epsilon M^{-1/\gamma}$.

Since the length of the intervals are bounded by strict inequality, $A_{\gamma,\epsilon}$ is open in $\mathcal{G}$. Also, by picking a suitable smooth function with small support one sees that $A_{\gamma,\epsilon}$ is non-empty. The main part of this note is devoted to showing that $A_{\gamma,\epsilon}$ is dense in $\mathcal{G}$, i.e.

Proposition 1. *$A_{\gamma,\epsilon}$ is open and dense in $(\mathcal{G}, d_{\mathcal{G}})$.*

Before proceeding to the proof let us draw some corollaries.

Set $\gamma = \alpha + \frac{1}{n}$ and $\epsilon = \frac{1}{n}$ in Proposition 1, we obtain the following, according to Baire category theorem.

Corollary 1. *$\bigcap_{n=1}^{\infty} A_{\alpha+\frac{1}{n},\frac{1}{n}}$ is a dense G_δ set in $(\mathcal{G}, d_{\mathcal{G}})$.*

Note that if $(E, \mu) \in \bigcap_{n=1}^{\infty} A_{\alpha+\frac{1}{n},\frac{1}{n}}$, then the lower Minkowski dimension $\underline{\dim}_M(E) \leq \alpha$, and in particular, $\dim_H(E) \leq \alpha$. On the other hand, if $\mu(\mathbb{T}) \neq 0$, then (2) implies $\dim_H(E) \geq \alpha$. Since the pairs (E, μ) with $\mu(\mathbb{T}) = 0$ form a subset of $\mathcal{G}$ whose complement is open and dense. We obtain the following

Theorem 1. *Quasi-all (E, μ) in $(\mathcal{G}, d_{\mathcal{G}})$ satisfy $\underline{\dim}_M(E) = \dim_H(E) = \alpha$.*

In particular, pick such a pair (E, μ) that is close to $(\mathbb{T}, \tau)$, and normalize μ, we obtain

Theorem 2. *Given $0 < \alpha < 1$, there exists a Borel probability measure μ on $\mathbb{T}$ supported on a compact set of Hausdorff and lower Minkowski dimension α, such that*

$$\lim_{|r|\to\infty} |r|^{\alpha/2}\hat{\mu}(r) = 0$$

and

$$\mu(I) \leq C|I|^{\alpha}$$

for all interval I. Moreover, one can make

$$\sup_{r\in\mathbb{Z}} |r|^{\alpha/2}|\hat{\mu}(r)| < \epsilon$$

where $\epsilon > 0$ is any prescribed number.

Now pick (E, μ) as in Theorem 1 which is close to $(\mathbb{T}, f\tau)$ where f is a suitable smooth function with small support, normalize μ, and then pass from $\mathbb{T}$ to $\mathbb{R}$ in the usual way (see e.g. [1], p. 252), we obtain the following.

Theorem 3. *Given $0 < \alpha < 1$, there exists a Borel probability measure μ on $\mathbb{R}$ supported on a compact set of Hausdorff and lower Minkowski dimension α, such that*

$$\lim_{|\xi|\to\infty} |\xi|^{\alpha/2}\hat{\mu}(\xi) = 0$$

and

$$\mu(I) \lesssim |I|^{\alpha}$$

for all interval I in $\mathbb{R}$.

We now turn to the proof of Proposition 1.

Given $(E, \mu) \in \mathcal{G}$, we first approximate it by measures with smooth densities. For this we use bump functions.

Lemma 2. *Let K be a nonnegative smooth function on $\mathbb{R}$ supported in $(-\pi/2, \pi/2)$ such that $\int_{\mathbb{R}} K(t)dt = 1$. If N is a positive integer, define $K_N(t) = NK(Nt)$. Regard K_N as a function defined on $\mathbb{T}$, then*

(i) $\int_{\mathbb{T}} K_N(t)dt = 1$

(ii) $|\hat{K}_N(r)| \lesssim (N/|r|)^L$

(iii) $\|K_N\|_\infty \lesssim N$

where the implicit constants depend only on K and the positive integer L.

From now on we fix such a bump function K.

Given a finite Borel measure μ, its convolution with K_N is defined to be

$$K_N * \mu(t) = \int_{\mathbb{T}} K_N(t-s)d\mu(s).$$

Note that $g_N = K_N * \mu$ is a smooth function, and $\hat{g}_N(r) = \hat{K}_N(r)\hat{\mu}(r)$.

Lemma 3. *Pairs of the form $(E, f\tau)$ where f is a nonnegative smooth function form a dense subset of $\mathcal{G}$.*

Proof. Given $(E, \mu) \in \mathcal{G}$, direct checking shows that $(E \cup \operatorname{supp}(g_N), g_N\tau)$ converges to (E, μ) in $\mathcal{G}$ as $N \to \infty$. We remark that here we have used (1) to show the convergence. □

Now by Lemma 3, given $(F, \sigma) \in \mathcal{G}$, it can be approximated by $(E, f\tau) \in \mathcal{G}$ where f is smooth. Easy checking shows that $(E, f\tau)$ can be further approximated by $(E, (1-\delta)f\tau)$ for small $\delta > 0$. Hence the proof of Proposition 1 reduces to the following.

Lemma 4. *If f is smooth and $(E, f\tau) \in \mathcal{G}$ satisfies*

$$\int_I f dt \leq (C-\delta)|I|^\alpha$$

for all interval I and some $0 < \delta < C$. Then $(E, f\tau)$ can be approximated by elements in $\mathcal{A}_{\gamma,\epsilon}$

Lemma 4 will follow from the following lemma.

Lemma 5. *Given $\eta > 0$ and $\theta > 0$, we can find $0 < \kappa < \theta$ and a nonnegative smooth function g_η with $\int_{\mathbb{T}} g_\eta(t)dt = 1$ having the following properties:*
(i) $|\hat{g}_\eta(r)| \leq \eta |r|^{-\alpha/2}$, for $r \neq 0$
(ii) $\int_I g_\eta(t)dt \leq (1+\eta)|I|/2\pi$, for $|I| \geq \kappa/2$
(iii) $\int_I g_\eta(t)dt \leq \eta |I|^\alpha$, for $|I| \leq \kappa$
(iv) We can find intervals $I_1, \cdots, I_M$ with

$$\operatorname{supp}(g_\eta) \subset \bigcup_{m=1}^{M} I_m$$

and $|I_1| = \cdots = |I_M| < \eta M^{-1/\gamma}$.

Proof of Lemma 4 from Lemma 5. Given $(E, f\tau)$ as in Lemma 4 and $\varepsilon > 0$, we shall choose η, θ in Lemma 5 small enough such that

$$(F \cup (E \cap \operatorname{supp}(g_\eta)), 2\pi g_\eta f\tau) \in \mathcal{A}_{\gamma,\epsilon}$$

and

$$d_{\mathcal{G}}\big((E, f\tau), (F \cup (E \cap \operatorname{supp}(g_\eta)), 2\pi g_\eta f\tau)\big) < \varepsilon$$

where g_η is the function described in Lemma 5 and F is a finite $\varepsilon/6$-net of E.

The fact that $(F \cup (E \cap \operatorname{supp}(g_\eta)), 2\pi g_\eta f\tau) \in \mathcal{G}$ is clear except (2). To show (2) we argue as follows. Since f is uniformly continuous, we can ensure that $|f(s) - f(t)| \leq \eta$ whenever $|s-t| \leq \kappa$, provided θ is small enough depending on η and f. Now any interval I of length at least κ can be written as the union of a collection of disjoint intervals J with $\kappa/2 \leq |J| \leq \kappa$. Thus using condition (ii) of Lemma 5,

$$\begin{aligned}
\int_I 2\pi f(t) g_\eta(t) dt &= \sum_J \int_J 2\pi f(t) g_\eta(t) dt \\
&\leq \sum_J \int_J \big(\frac{1}{|J|}\int_J f(s)ds + \eta\big) 2\pi g_\eta(t) dt \\
&= \sum_J \big(\frac{1}{|J|}\int_J f(s)ds + \eta\big) \int_J 2\pi g_\eta(t) dt \\
&\leq \sum_J \big(\frac{1}{|J|}\int_J f(s)ds + \eta\big)(1+\eta)|J|
\end{aligned}$$

$$
\begin{aligned}
&= \sum_J (1+\eta) \int_J f(s)ds + \eta(1+\eta)|J| \\
&= (1+\eta) \int_I f(t)dt + \eta(1+\eta)|I| \\
&\le (1+\eta)(C-\delta)|I|^\alpha + \eta(1+\eta)C|I|^\alpha \\
&\le C|I|^\alpha
\end{aligned}
$$

provided that η is small enough.

If $|I| \le \kappa$, condition (iii) of Lemma 5 yields

$$\int_I 2\pi f(t) g_\eta(t) dt \le \|f\|_\infty 2\pi \int_I g_\eta(t) dt \le \|f\|_\infty 2\pi\eta |I|^\alpha \le C|I|^\alpha$$

provided that η is small enough.

Denote $\sigma = g_\eta \tau$, then condition (ii) and (iii) of Lemma 5 implies that $\sigma(I) \lesssim |I|^\alpha$ for all interval I, where the implicit constant depends only on α. By condition (iv) of Lemma 5,

$$1 = \sigma(\operatorname{supp}(g_\eta)) \le \sum_m \sigma(I_m) \lesssim \sum_m |I_m|^\alpha < \eta^\alpha M^{1-\frac{\alpha}{\gamma}},$$

which implies $M \to \infty$ as $\eta \to 0$. In particular, $M \ge \#F$ provided η is small enough.

Now if $\eta < 2^{-1/\gamma}\epsilon$, then $\eta M^{-1/\gamma} < \epsilon(2M)^{-1/\gamma}$. On the other hand, we can of course find intervals $I_{M+1}, \cdots, I_{2M}$ of length $|I_1|$ that cover F since $M \ge \#F$. Collecting these intervals together with $I_1, \cdots, I_M$ we get $2M$ intervals of length $|I_1| < \epsilon(2M)^{-1/\gamma}$ such that

$$F \cup (E \cap \operatorname{supp}(g_\eta)) \subset \bigcup_{m=1}^{2M} I_m.$$

This shows $(F \cup (E \cap \operatorname{supp}(g_\eta)), 2\pi g_\eta f \tau) \in \mathcal{A}_{\gamma,\epsilon}$.

Finally, we turn to bound $d_{\mathcal{G}}$. Since F is a $\varepsilon/6$-net of E, it follows immediately that $d_{\mathcal{F}}(E, F \cup (E \cap \operatorname{supp}(g_\eta))) < \varepsilon/3$. So it remains to show that

$$|\widehat{2\pi g_\eta f}(0) - \hat{f}(0)| < \varepsilon/3$$

and

$$\sup_{r \in \mathbb{Z}} |r|^{\alpha/2} |\widehat{2\pi g_\eta f}(r) - \hat{f}(r)| < \varepsilon/3$$

provided η is small enough. We only show the first one, the second one is similar.

$$
\begin{aligned}
|\hat{f}(0) - \widehat{2\pi g_\eta f}(0)| &= |\hat{f}(0) - \sum_{u \in \mathbb{Z}} \hat{g}_\eta(u) \hat{f}(-u)| \\
&= |\sum_{u \neq 0} \hat{g}_\eta(u) \hat{f}(-u)|
\end{aligned}
$$

$$\lesssim \sum_{u\neq 0} \eta |u|^{-\alpha/2}|u|^{-1}$$
$$\lesssim \eta < \varepsilon/3$$

provided that η is sufficiently small. □

We now turn to the proof of Lemma 5. The following proof, inspired by a paper of Kaufman [2], makes a clever use of the prime numbers.

Write $\mathcal{P}(N)$ for the set of primes p with $N+1 \leq p \leq 2N$. We shall need the following (weaker) version of the prime number theorem.

Lemma 6. *There exists constants* $0 < A < 1 < B$ *such that*

$$A\frac{N}{\log N} \leq \#\mathcal{P}(N) \leq B\frac{N}{\log N}$$

for all $N \geq 2$.

We combine this with a simple observation.

Lemma 7. *If* $m \geq 2$ *and*

$$\sigma_m = \sum_{j=1}^{m-1} \delta_{2\pi j/m},$$

then

$$\hat{\sigma}_m(r) = \begin{cases} m-1, & \text{if } r \equiv 0 \pmod{m} \\ -1, & \text{otherwise.} \end{cases}$$

For $N \geq 2$, we set $q(N) = \sum_{p\in\mathcal{P}(N)}(p-1)$ and

$$\tau_N = \frac{1}{q(N)} \sum_{p\in\mathcal{P}(N)} \sigma_p,$$

The following lemma gives the key properties of τ_N.

Lemma 8. *Suppose* $N \geq 2$*, then*

(i) If $p, q \in \mathcal{P}(N)$ *and* $p \neq q$*, then*

$$2\pi u/p \neq 2\pi v/q$$

whenever $1 \leq u \leq p-1$ *and* $1 \leq v \leq q-1$.

(ii) The measure τ_N *is a probability measure of the form*

$$\tau_N = \frac{1}{\#E(N)} \sum_{e\in E(N)} \delta_e$$

with $E(N)$ *a finite set with the property that*

$$|e-f| \geq 2\pi N^{-2}$$

whenever $e, f \in E(N)$ *and* $e \neq f$.

(iii) $\tau_N(I) \leq |I|/2\pi + 2/N$ *for all interval* I.

(iv) $AN^2/\log N \leq \#E(N) \leq 2BN^2/\log N$

(v) Let k be a positive integers, then

$$|\hat{\tau}_N(r)| \leq 4A^{-1}kN^{-1}\log N$$

for all $1 \leq |r| \leq N^k$.

We are now ready to prove Lemma 5 by convolving τ_N with suitable bump functions.

Proof of Lemma 5. Consider τ_N as in Lemma 8. We choose M to be the number of points in the support of τ_N. Recall from Lemma 8 that

$$AN^2/\log N \leq M \leq 2BN^2/\log N.$$

Provided that N is sufficiently large, we can find an integer P with

$$\frac{1}{4}\eta M^{-1/\gamma} \leq P^{-1} \leq \frac{1}{2}\eta M^{-1/\gamma}.$$

Set $g_\eta = K_P * \tau_N$ and $\kappa = 4\pi\eta^{-1}N^{-1}$. Clearly, $\kappa < \theta$ if N is large enough. We show that g_η satisfies the properties in Lemma 5 if N is chosen large enough.

It is easy to see that $\int_{\mathbb{T}} g_\eta(t)dt = 1$ and that condition (iv) is satisfied. To prove (i), we choose $\beta \in (\alpha, \gamma)$ and notice that for $1 \leq |r| \leq N^{2/\beta}$,

$$\begin{aligned}
|\hat{g}_\eta(r)| &= |\hat{K}_P(r)||\hat{\tau}_N(r)| \\
&\leq |\hat{\tau}_N(r)| \\
&\lesssim N^{-1}\log N \\
&= |r|^{-\alpha/2}\frac{|r|^{\alpha/2}}{N^{\alpha/\beta}}\frac{N^{\alpha/\beta}\log(N)}{N} \\
&\leq |r|^{-\alpha/2}\frac{\log(N)}{N^{1-\alpha/\beta}} \\
&\leq \eta|r|^{-\alpha/2}
\end{aligned}$$

provided that N is large enough.

On the other hand, choose integer L such that $L(1-\beta/\gamma) \geq 2$. Then for $|r| \geq N^{2/\beta}$,

$$\begin{aligned}
|\hat{g}_\eta(r)| &= |\hat{K}_P(r)||\hat{\tau}_N(r)| \\
&\leq |\hat{K}_P(r)| \\
&\lesssim (\frac{P}{|r|})^L \\
&\lesssim (\frac{\eta^{-1}M^{1/\gamma}}{|r|})^L \\
&\lesssim (\frac{\eta^{-1}N^{2/\gamma}}{|r|})^L \\
&\lesssim (\frac{\eta^{-1}|r|^{\beta/\gamma}}{|r|})^L
\end{aligned}$$

$$
\begin{aligned}
&= \eta^{-L}|r|^{(\beta/\gamma-1)L} \\
&\lesssim \eta^{-L}|r|^{-\alpha/2}|r|^{-1} \\
&\lesssim \eta^{-L}|r|^{-\alpha/2}N^{-2/\beta} \\
&\leq \eta|r|^{-\alpha/2}
\end{aligned}
$$

provided that N is large enough. This proves condition (i) in Lemma 5.

To prove (ii), notice that for $|I| \geq \kappa$,

$$
\begin{aligned}
\int_I g_\eta(t)dt &= \int_{\mathbb{T}} g_\eta(t)\chi_I(t)dt \\
&= \int_{\mathbb{T}}\int_{\mathbb{T}} K_P(t-s)d\tau_N(s)\chi_I(t)dt \\
&= \int_{\mathbb{T}}\int_{\mathbb{T}} K_P(t-s)\chi_I(t)dtd\tau_N(s) \\
&= \int_{\mathbb{T}}\int_{\mathbb{T}} K_P(t)\chi_I(t+s)dtd\tau_N(s) \\
&= \int_{\mathbb{T}}\int_{\mathbb{T}} \chi_I(t+s)d\tau_N(s)K_P(t)dt \\
&= \int_{\mathbb{T}} \tau_N(I-t)K_P(t)dt \\
&\leq \int_{\mathbb{T}} (|I|/2\pi + 2/N)K_P(t)dt \\
&= |I|/2\pi + 2/N \\
&= |I|/2\pi + \eta\kappa/2\pi \\
&\leq |I|/2\pi + \kappa^{-1} \\
&\leq (1+\eta)|I|/2\pi
\end{aligned}
$$

Finally, the proof of condition (iii) splits into three parts depending on the length of the interval I. First suppose $\pi N^{-2} \leq |I| \leq \kappa$. Lemma 8 (ii) tells us that

$$\#(I \cap E_N) \leq N^2|I| + 1$$

and so

$$\tau_N(I) \leq (N^2|I|+1)M^{-1}$$

for all interval I. By similar argument as above,

$$
\begin{aligned}
\int_I g_\eta(t)dt &\lesssim (N^2|I|+1)M^{-1} \\
&\lesssim (\log N)(|I| + N^{-2}) \\
&\lesssim (\log N)|I| \\
&= (\log N)|I|^{1-\alpha}|I|^{\alpha} \\
&\leq (\log N)\kappa^{1-\alpha}|I|^{\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&= (\log N)(4\pi\eta^{-1}N^{-1})^{1-\alpha}|I|^{\alpha} \\
&\le \eta|I|^{\alpha}
\end{aligned}
$$

provided that N is large enough. $\square$

If $M^{-1/\gamma} \le |I| \le \pi N^{-2}$, then I covers at most one point of $E(N)$. Hence

$$\tau_N(I) \le M^{-1}$$

and so

$$
\begin{aligned}
\int_I g_\eta(t)dt &\le M^{-1} \\
&\le |I|^{\gamma} \\
&= |I|^{\gamma-\alpha}|I|^{\alpha} \\
&\le (\pi N^{-2})^{\gamma-\alpha}|I|^{\alpha} \\
&\le \eta|I|^{\alpha}
\end{aligned}
$$

provided that N is large enough.

If $|I| \le M^{-1/\gamma}$, then noticing that

$$\|g_\eta\|_\infty \le M^{-1}\|K_P\|_\infty,$$

we have

$$
\begin{aligned}
\int_I g_\eta(t)dt &\le |I|\|g_\eta\|_\infty \\
&\lesssim |I|M^{-1}P \\
&\lesssim |I|M^{-1}\eta^{-1}M^{1/\gamma} \\
&\lesssim |I|\eta^{-1}|I|^{\gamma-1} \\
&= \eta^{-1}|I|^{\gamma-\alpha}|I|^{\alpha} \\
&\le \eta^{-1}M^{-(\gamma-\alpha)/\gamma}|I|^{\alpha} \\
&\le \eta|I|^{\alpha}
\end{aligned}
$$

provided that N is large enough. This concludes the proof.

References

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[2] R. Kaufman, *On a theorem of Jarník and Besicovitch,* Acta Arith. 39 (1981), 265–267.

[3] T. W. Körner, *Hausdorff and Fourier dimension,* Studia Math. 206 (2011), no. 1, 37–50.